Problem-Based Learning of Mathematics

In a problem-based mathematics lesson, the bulk of the cognitive work of learning is done by the students, with the teacher orchestrating productive interactions between mathematics and learners. Typically, a problem-based math lesson begins with the introduction of an appropriately challenging "group-worthy" problem or task, one that has multiple entry points, requires students to actively *grapple,* and supports collaborative problem solving (Lotan, 2003; Hiebert & Grouws, 2007). The purpose of the launch is to *engage* the students in the mathematics central to the task and to *promote access* to the math for all students (Lampert, 2001; Horn, 2012). Promoting access for all students must be balanced, however, with the need to establish and maintain the cognitive demand of the task (Stein et al., 2000).

In a high-functioning problem-based math lesson, the students first engage the task individually to develop a mind-hold on the relevant mathematics and then come together in pairs or small groups to share perspectives on the problem and to work collaboratively toward a solution (Slavin, 1991). The teacher's role during this investigation/inquiry phase of the lesson is to *listen to* student solution attempts rather than *listening for* particular answers and to ask probing questions that prompt more effective problem solving and collaboration (Fennema, Carpenter, & Peterson, 1989). The teacher's questions should *focus* attention on important aspects of the problem, without *funneling* students into a particular solution strategy (NCTM, 2014). This process is enhanced when teachers have made an effort (in advance) to anticipate how students might respond to the task at hand, considering questions they might ask to probe and support connections, build on prior knowledge, and promote student understanding around the desired learning goal(s) for the lesson (Smith & Stein, 2011; NCTM, 2014).

An important aspect of the teacher's facilitation of the investigation phase is to "manage the clock" – too much time allocated to this phase of the lesson may decrease the sense of urgency that promotes successful collaboration, too little time undervalues student struggle and reasoning. The strategic use of distributed shares serves to promote expanded access to evolving ideas and processes and ideally results in more productive grappling during the problem solving endeavor (Marzano, 2001; NCTM 2014). Whole class discussions provide a forum for the teacher to underscore the value of student ideas and explanations rather than simply problem resolution. As students develop solution strategies, they are called upon to construct viable arguments and listen and respond to one another's mathematical argumentation (CCSS, 2009). In some instances, the teacher may provide extensions to the problems for students who have solved the initial task and are ready to probe the mathematics more deeply.

Finally, the teacher has the responsibility to help students make explicit connections between key mathematical ideas and to promote as much of a consolidation of the learning as possible (Ball 1993; Lampert, 2001; Boaler & Humphries, 2005; Hiebert & Grouws 2007). This usually involves the teacher prompting carefully selected groups to share their solution strategies in front of the whole class. The teacher is also mindful of sequencing these student presentations in the order that would seem to maximize the learning goals for that lesson (Smith & Stein, 2011). A particularly challenging aspect of this phase of the lesson is to promote a genuine and productive interaction between student presenters and the other students in the class. Instead of the teacher simply summarizing what she believes students have learned, the students come to rely on each other as mathematical authorities, understanding each other's mathematical reasoning becomes the ultimate arbiter for sense-making in the mathematics classroom (Webel, 2010).

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